Real Time Disparity Estimation

with Variational Methods

Sergey Kosov, Thorsten Thormählen, Hans-Peter Seidel

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Overview

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I. Introduction
- Disparity Estimation
- Methods Overview

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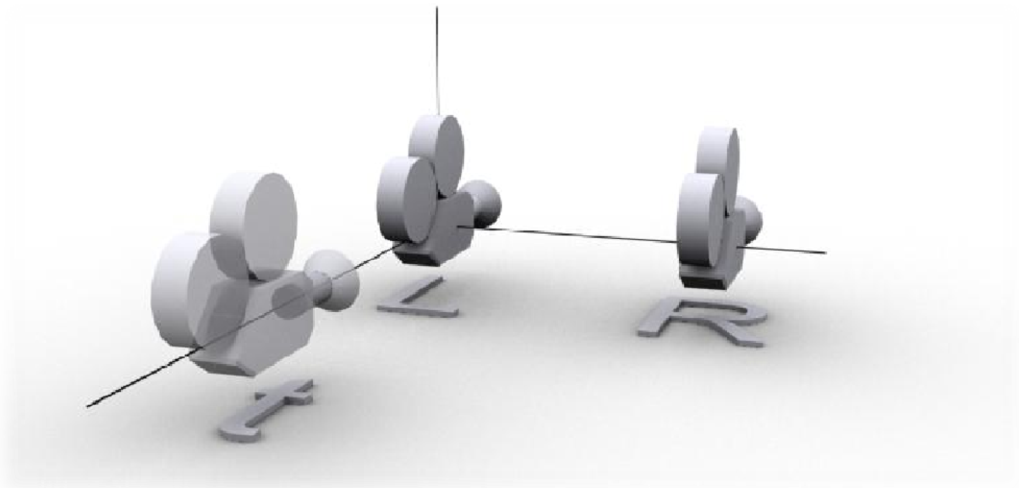
- Euler-Lagrange Equation
- Full Multi Grid
- Multi Level Adaptation Technique

III. Results
- Speed / Quality Discussion
- Future Work

Introduction



- **Disparity field** is a vector field of pattern of apparent motion of objects, surfaces, and edges in a visual scene caused by the relative motion between an observer (eyes or cameras) and the scene.

Disparity Field

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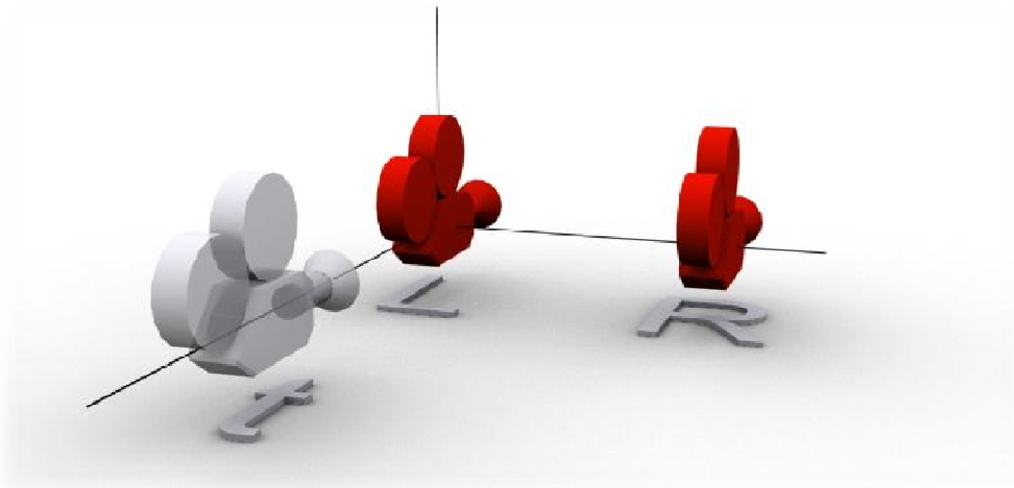
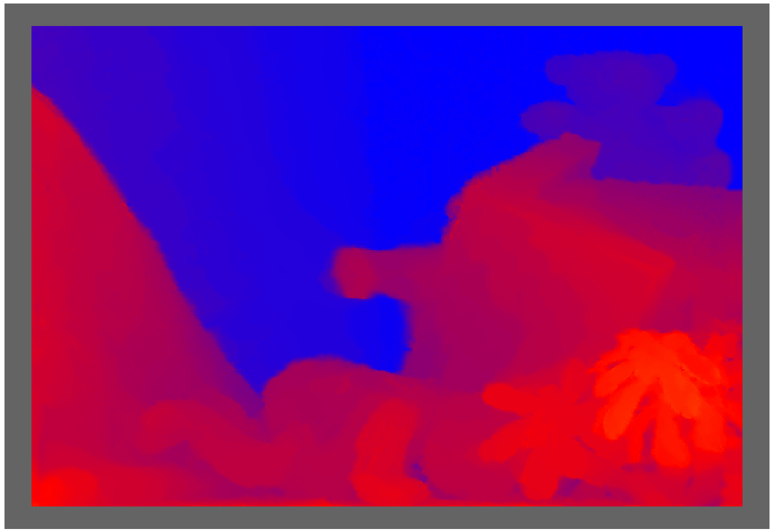
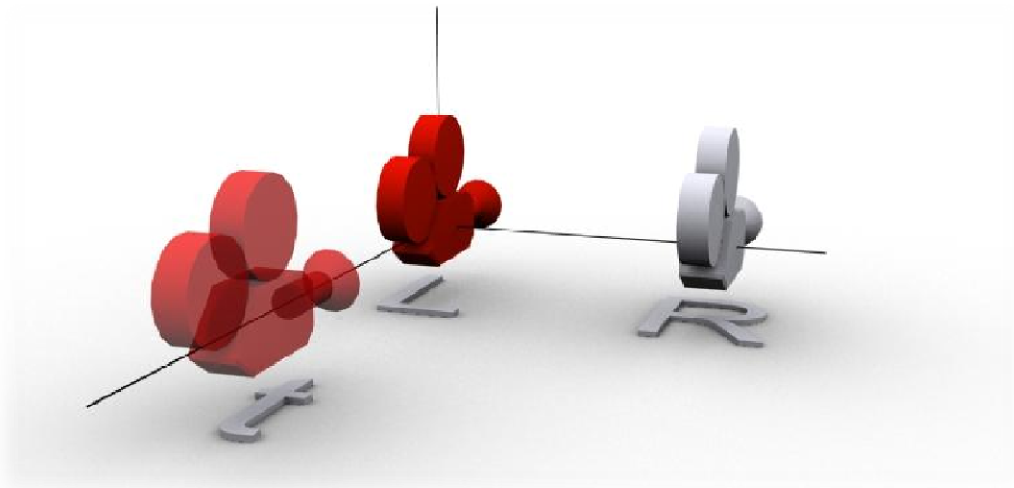
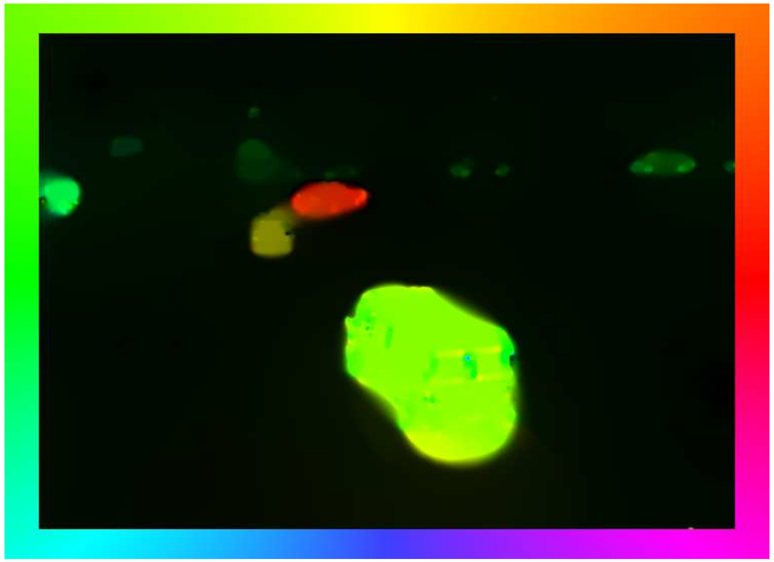




Optical Flow Field





Methods Overview

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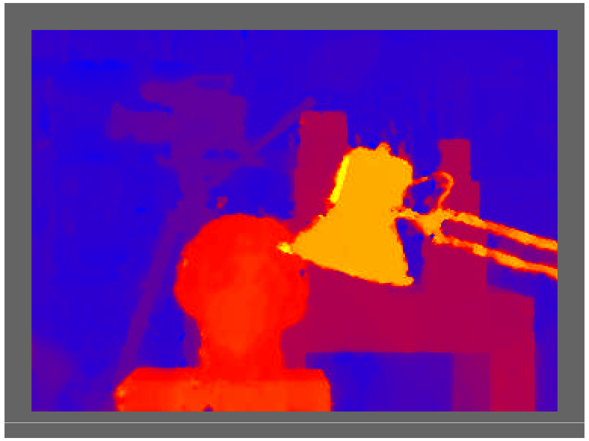
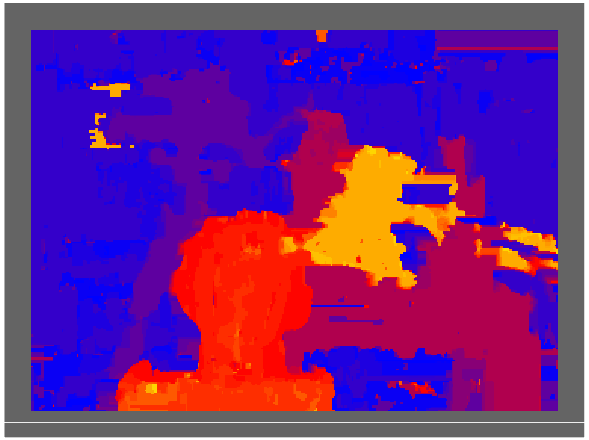
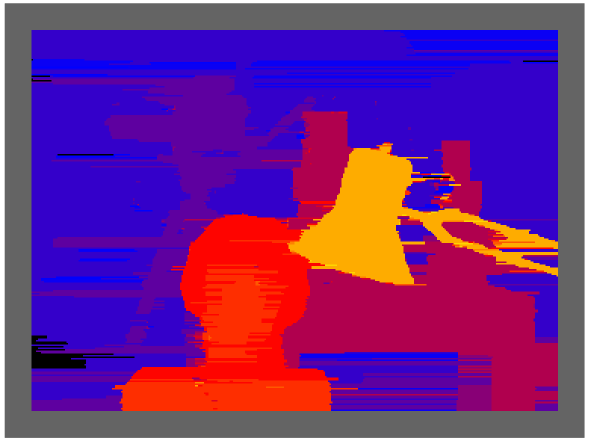


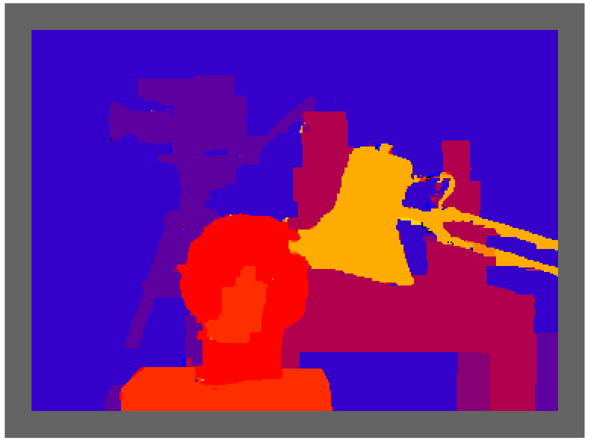
Ours: Variational Method



Infection Method



Belief Propagation Method



Graph Cuts Method

Euler-Lagrange Equation

- *Constancy assumption* on the grey value:

$$I_1\left(x + z(x, y), y\right) = I_2\left(x, y\right)$$

Euler-Lagrange Equation

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- *Constancy assumption* on the grey value:

$$I_1(x+z(x,y),y)=I_2(x,y)$$

- Data term:

$$Arg\ \min \left\| I_1(x+z(x,y),y)-I_2(x,y) \right\|_2$$

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- Energy Functional:

$$E(z(x,y))=\iint_{\Omega}\left\| I_1(x+z(x,y),y)-I_2(x,y)\right\|^2 dS$$

Euler-Lagrange Equation

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- Energy Functional:

$$E(z(x,y))=\iint_\Omega \left\| I_1(x+z(x,y),y)-I_2(x,y)\right\|^2 dS$$

- Preserving discontinuities:

$$E(z(x,y))=\iint_\Omega \left\| I_1(x+z(x,y),y)-I_2(x,y)\right\|^2+\varphi\cdot\Psi\left(\left|\nabla z(x,y)\right|^2\right)dS$$

  - Where $\Psi\left(s^2\right)=\lambda\sqrt{\lambda^2+s^2}-\lambda^2$ is non-linear regularization function

Euler-Lagrange Equation

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- The mathematicians *Leonard Euler* (1707-1783) and *Joseph-Louis Lagrange* (1736-1813) belong to the founders of the calculus of variations.

- Steady state of a function:

  $$f = f(x)$$

- Satisfies the equation:

  $$f_x(x) = 0$$

  - where $f_x(x) \equiv \dfrac{df(x)}{dx}$

- Solution:

  $$x = a$$

- Steady state of a functional

  $$E(f(x)) = \int_l F(x, f(x), f_x(x))dx$$

- Satisfies the equation:

  $$\frac{\partial F}{\partial f} - \frac{d}{dx}\frac{\partial F}{\partial f_x} = 0$$

- Solution:

  $$f(x) = \varphi(x)$$

Euler-Lagrange Equation

- A smooth function $z(x, y)$ that minimizes the energy functional:

$$E(z(x,y)) = \iint_{\Omega} F\left(x, y, z(x,y), \frac{\partial z(x,y)}{\partial x}, \frac{\partial z(x,y)}{\partial y} \right) dS$$

- Satisfies necessary the Euler-Lagrange equation:

$$\frac{\partial F}{\partial z} - \frac{d}{dx}\frac{\partial F}{\partial z_x} - \frac{d}{dy}\frac{\partial F}{\partial z_y} = 0$$

  - where $z_u = \dfrac{\partial z}{\partial u}$ and $z_v = \dfrac{\partial z}{\partial v}$.

Multi Grid

- In our case we can write the Euler-Lagrange equation:

$$I_{1x}(x+z,y)\cdot(I_1(x+z,y)-I_2(x,y))-\varphi\cdot div\left(\Psi'\left(|\nabla z|^2\right)\nabla z\right)=0$$

Multi Grid

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$$I_{1x}(x+z,y)\cdot(I_1(x+z,y)-I_2(x,y))-\varphi\cdot div\left(\Psi'\left(|\nabla z|^2\right)\nabla z\right)=0$$

- Which could be rewritten in form:

$$A_1 z + A_2 - \varphi\cdot\Lambda(z) = 0$$

  - where A_i are linear operators,
  - and $\Lambda(z)$ is a *non-linear* deferential operator.

Multi Grid

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- Which could be rewritten in form:

$$A_1 z + A_2 - \varphi\cdot\Lambda(z) = 0$$

  - where A_i are linear operators,
  - and $\Lambda(z)$ is a *non-linear* differential operator.
- We reformulate this non-linear equation system:

$$L(x) = f$$

  - where x denotes vector $(u^T, v^T)^T$,
  - L is a *non-linear* operator,
  - f stands for the right hand side.

Full Approximation Scheme

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- Multigrid approach was offered by R. Fedorenko in 1961
- Very effective for solving elliptic and parabolic PDE systems

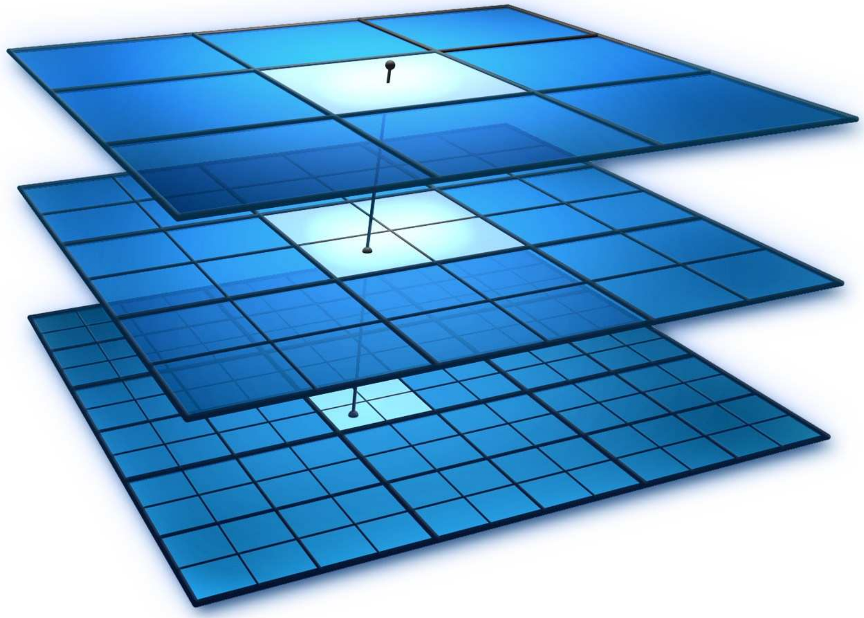
- Uses a set of grids with different step sizes:

$$G(h) = (\Delta h, \Delta h)$$

$$G(H^i) = G(2^i h)$$

with approximation to a grid

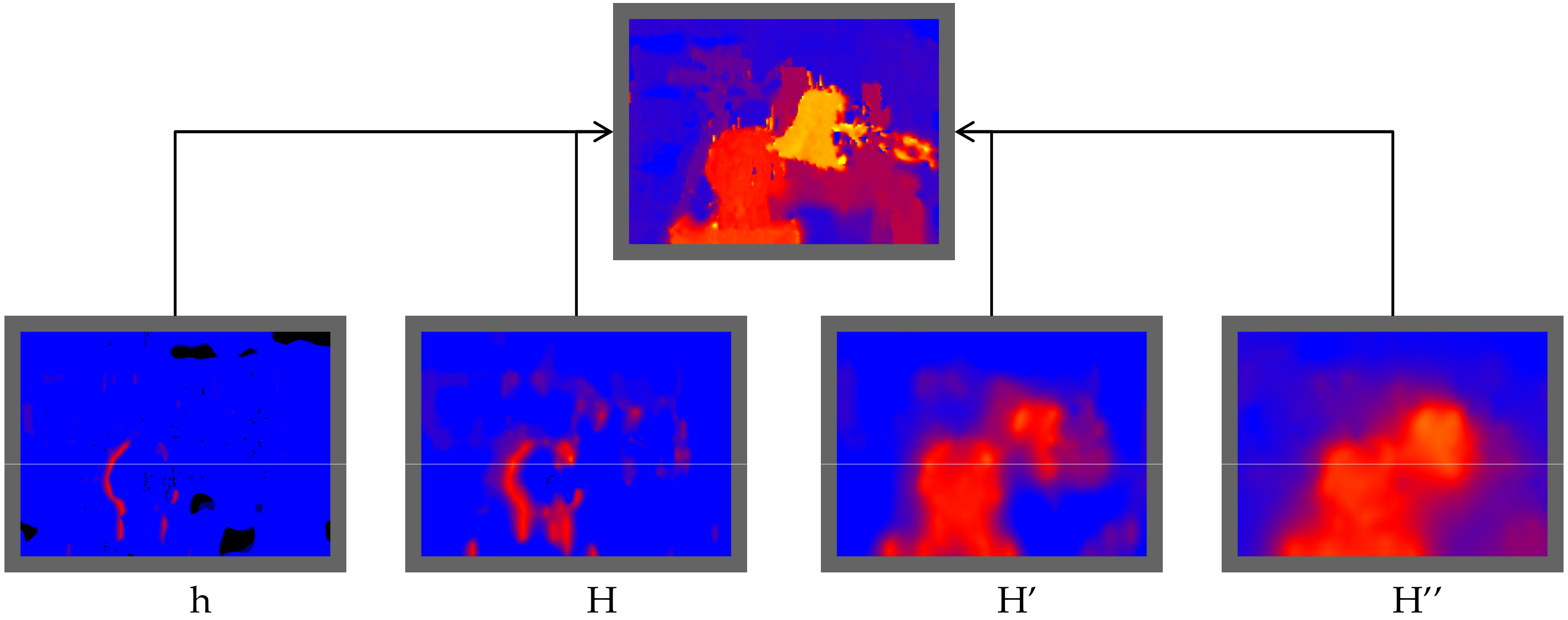
$$L^h x^h = f^h$$



Full Approximation Scheme

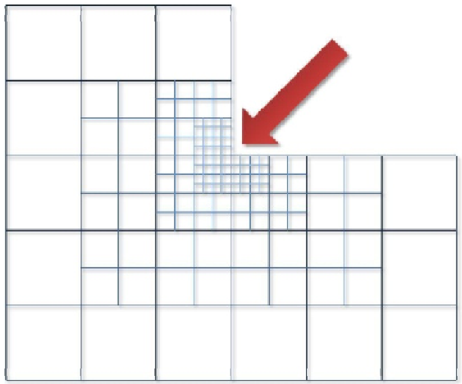
- First way
  - Solve problem on a given grid by calling Multigrid on a coarse approximation to get a good guess to refine
- Second way
  - Think of error as a sum of sine curves of different frequencies
  - Same idea as FFT solution, but not explicit in algorithm
  - Each call to Multigrid responsible for suppressing coefficients of sine curves of the lower half of the frequencies in the error

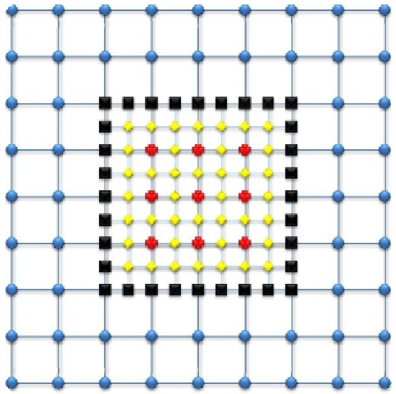
Full Multigrid

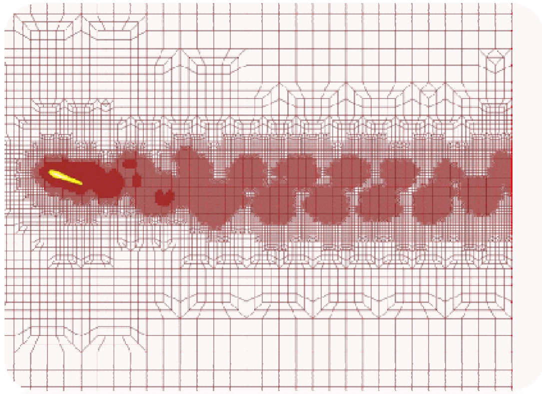


Multilevel Adaptation Technique

- Adaptive grids
  - Increased computational savings over a static grid approach
  - Effective multi grid and adaptive grid combination
  - Easy and effective parallelizable
- Irregular grids
  - Concentrate nodes near to a local peculiarity
  - Simple combination of two grid levels







Results

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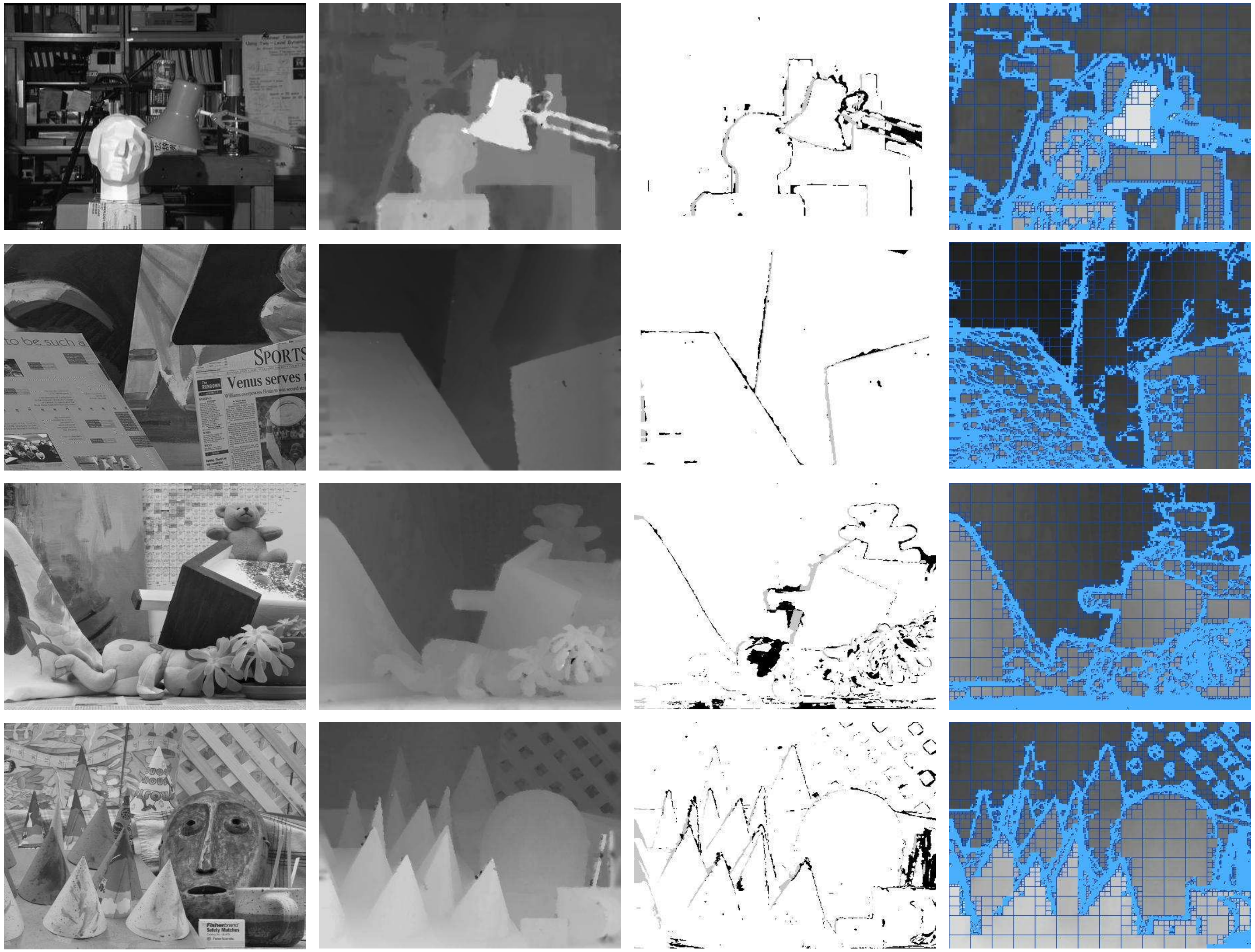

Tsukuba
Bad pixels: *3,33%*
Time: *465 ms*

Venus
Bad pixels: 1,*15%*
Time: 265 *ms*

Teddy
Bad pixels: 5,*88%*
Time: *305 ms*

Cones
Bad pixels: *4,61%*
Time: 285 *ms*

Results

- Excerpt from the evaluation table generated by the Middlebury stereo evaluation webpage (error threshold = 2 pixels).

Algorithm	Tsukuba			Venus			Teddy			Cones			Average percent
	nonocc	all	disc	nonocc	all	disc	nonocc	all	disc	nonocc	all	disc	of bad pixels
DoubleBP	*0,83*	*1,24*	*4,49*	*0,10*	*0,35*	*1,46*	*1,41*	*4,13*	*4,73*	*1,71*	*7,20*	*5,16*	*2,72*
CoopRegion	*0,77*	*1,00*	*4,14*	*0,11*	*0,18*	*1,53*	*2,14*	*3,41*	*6,61*	*2,10*	*5,95*	*6,24*	*2,80*
…													
Ours: RealTimeVar	*1,33*	*3,13*	*6,94*	*0,27*	*1,07*	*3,16*	*1,30*	*2,30*	*3,87*	*2,31*	*3,43*	*6,90*	***3,00***
…													
MultiResGC	*0,67*	*1,05*	*3,64*	*0,22*	*0,46*	*2,97*	*4,20*	*7,13*	*11,6*	*3,22*	*8,80*	*8,07*	*4,30*
RealTimeBP	*1,25*	*3,04*	*6,66*	*0,63*	*1,53*	*7,68*	*5,68*	*8,27*	*10,2*	*2,90*	*9,11*	*8,27*	*5,43*
RealTimeGPU	*1,34*	*3,27*	*7,17*	*1,02*	*1,90*	*12,4*	*3,90*	*8,65*	*10,4*	*4,37*	*10,8*	*12,3*	*6,46*
Infection	*6,34*	*7,81*	*22,8*	*2,70*	*3,66*	*26,0*	*12,8*	*18,3*	*33,5*	*10,7*	*16,6*	*30,1*	*15,9*

Future Work

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- Main contributions so far:
  - The most accurate disparity estimation method among real-time methods *(according to Middlebury University web-site)*
  - Effective combination of multigrids with grid adaptation techniques
  - Multigrid powered by *null*-cycles

- Future work:
  - From the disparity estimation to the pure optical flow reconstruction
  - User recognition and authentication with stereo web-cams
  - Use your body as an input device in interactive games
  - Robots with stereo cameras as eyes

The end

- Thank you for your attention

Ready to answer your questions ☺