Computer Graphics Worksheet
Ray-Geometry Intersection Algorithms

Dr. Sergey Kosov
Jacobs University Bremen

Problem 1. Points

Let $\overrightarrow{a}, \overrightarrow{b}, \overrightarrow{c}$ be points in $\mathbb{R}^3$.

a) Calculate the length of three sides of the triangle with vertices $\overrightarrow{a} = (1, -1, 2)^\top$, $\overrightarrow{b} = (3,3,8)^\top$ and $\overrightarrow{c} = (2,0,1)^\top$.

b) Using cosine law, show that the triangle from (a) has a right angle.

c) Find the angle α adjacent to vertex $\overrightarrow{a}$ in the triangle with vertices $\overrightarrow{a} = (2, -1, -1)^\top$, $\overrightarrow{b} = (0,1,-2)^\top$ and $\overrightarrow{c} = (1, -3, 1)^\top$.

Problem 2. Vectors

Given two vectors $\overrightarrow{u}, \overrightarrow{v}$ of length 1, provide two versions of a formula computing a vector $\vec{t}$ that is perpendicular to $\overrightarrow{u}$ and lying on the uv-plane. Both versions can contain vector addition and subtraction, and…

a) The first version of the formula should consist of only cross products.

b) The second version of the formula should consist only of dot products.

Provide geometric interpretation of these formulas

Problem 3. Triangle primitive

A triangle T is defined by its 3 vertices $\overrightarrow{a}, \overrightarrow{b}, \overrightarrow{c}$.

a) Compute the barycentric coordinates of the center of mass of T

b) Compute the barycentric coordinates of the incenter of T (center of the inscribed circle)

Problem 4. Ray-Surface Intersection

Given a ray $\vec{r}(t) = \vec{o} + t\,\vec{d}$ with origin $\vec{o} = (o_x, o_y, o_z)^\top$ and direction $\vec{d} = (d_x, d_y, d_z)^\top$, derive the equations to compare the parameter t for the intersection point(s) of the ray and the following implicitly represented surfaces:

a) An infinite plane $\left(\vec{p} - \vec{a}\right) \cdot \vec{n} = 0$ through point $\vec{a} = (a_x, a_y, a_z)^\top$ with surface normal $\vec{n} = (n_x, n_y, n_z)^\top$, where any point $\vec{p} = (x, y, z)^\top$ that satisfies the equation lies on the surface.

b) A sphere $\left(\vec{p} - \vec{c}\right) \cdot \left(\vec{p} - \vec{c}\right) = r^2$ with center $\vec{c} = (c_x, c_y, c_z)^\top$, radius $r \in \mathbb{R}$ where any point $\vec{p} = (x, y, z)^\top \in \mathbb{R}^3$ that satisfies the equation lies on the surface.

c) A quadric $ax^2 + by^2 + cz^2 + dxy + exz + fyz + gx + hy + iz + j = 0$.

- For this task you may want to represent the ray equation if form:

  $x = o_x + td_x$

  $y = o_y + td_y$

  $z = o_z + td_z$

  and then solve it for t
- Derive the ray-sphere intersection formula from it, as a special case

Problem 5. Reflection Rays

Given a ray $\vec{r}(t) = \vec{o} + t\,\vec{d}$ which hits a reflective surface at $t = t_{hit}$. The surface has the geometry normal $\vec{n}$ at the hit point. Assume that both, the ray direction $\vec{d}$ and the surface normal $\vec{n}$ are normalised. Compute the ray $\vec{I}(t)$ that has been reflected (assuming a perfect mirror reflection) by the surface.

Problem 6. Snell's Law

1. Let us consider a 2-dimensional slice through a 2-layer dielectric material such that the half space of positive y coordinates lies in a medium where light travels at constant speed c_a and the half-space of negative y coordinates in a medium where light travels at constant speed c_b. Assuming that light travels between the 2 points $P_a(2{,}3)$ and $P_b(-1, -2)$ by crossing the interface between the two media at some point $P_i(x_i{,}0)$, write the expression for the time of travel as a function of x_i, c_a and c_b.
2. According to Fermat's principle, light always travels between 2 points along the path with minimal time of travel. Write the equation that x_i must satisfy for the path taken between P_a and P_b to be valid.
3. Use the formulations you have computed above to re-derive Snell's law, which relates the refractive indices n_a and n_b to the angles of incidence and exitance of the light rays.