Computer Graphics Worksheet
Rendering Equation and BRDF

Dr. Sergey Kosov
Jacobs University Bremen

Problem 1. Analytical integration

Please calculate the surface area of a sphere (or, a hemisphere) with radius r with integrals:

a) using Cartesian coordinates
b) using spherical coordinates
c) explain with your own words the Jacobian, you used in (b) for transforming from cartesian to spherical domain of integration

Problem 2. Monte-Carlo integration

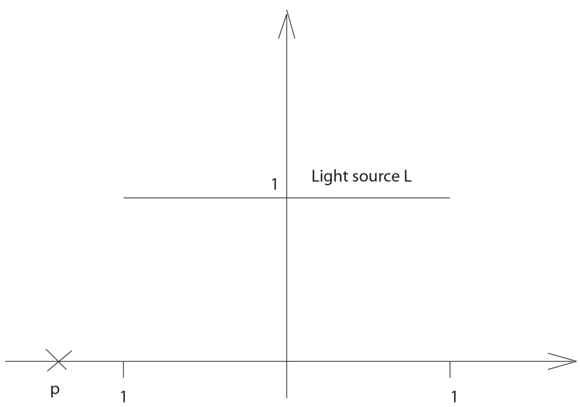
Please implement an algorithm of numerical integration to solve problem 1 (a) or (b).

a) using deterministic integration
b) using stochastic integration. Here your algorithm should use uniform distributed random samples $(\zeta, \xi) \in [0,1]$ (uniform sampling).
c) explain with your own words the advantage of Monte-carlo integration over the deterministic methods of integration.

Problem 3. Analytical solution of the rendering equation in 2D

The image below shows a simple 2D scene with a linear light source which reaches from -1 to 1 at y-position 1 and has uniform radiance 1 for each point and direction. Assume that the light source absorbs all light hitting it. Located at the $y = 0$ line is a lambertian material with the following BRDF:

$$f_r(x, \omega_o, \omega_i) = \frac{1}{\pi}$$



a) The standard rendering equation in 2D is given by:

$$L(x, \omega_o) = L_e(x, \omega_o) + \int_0^{\pi} f_r(x, \omega_o, \omega_i) \cdot L(x, \omega_i) \ \cos\theta_i \ d\omega_i$$

Solve the rendering equation analytically for each point $x = (p, 0)$ and direction ω_o. As the rendering equation shows, you have to integrate over the hemicircle of the points.

b) Let point $p = (x, 0)$ lie at the Lambertian surface and point $s = (y, 1)$ lie at the light source. The standard rendering equation in 2D is given by:

$$L(p, \omega_o) = L_e(p, \omega_o) + \int_{-1}^{1} f_r(p, \omega_o, \omega_i) \ L(s, -\omega_i) \ \frac{\cos\theta_i \cos\theta_o}{|p - s|^2} \ dy$$

Solve the rendering equation analytically for each point $p = (x, 0)$ integrating over the light source.

Hint 1: $\frac{d}{dx} \arctan x = \frac{1}{1 + x^2}$

Hint 2: $\int u dv = uv - \int v du$

Problem 4. BRDF

$$L_o(x, \omega_o) = L_e(x, \omega_o) + \int_{\Omega+} f(x, \omega_i, \omega_o) \ L_i(x, \omega_i) \ \cos\theta_i \ d\omega_i$$

Please implement an algorithm which computes an approximation of this equation. Assuming only point lights are used, describe the approximation that is performed. Explain in detail what is being approximated and give a formula for $L_o(x, \omega_o)$ that describes what is really computed, given:

- N, the number if lights,
- f, the BRDF at point x,
- $\vec{n}$, the normal at point x,
- $V(x, y) = \begin{cases} 1 & \text{if } x \text{ is visible from } y \\ 0 & \text{otherwise} \end{cases}$, the visibility function,
- y_i, the position of the light source i,
- I_i, the intensity of the **point light** i

Note: You can write the direction pointing from the point x to the point y as $x \to y$